Class XI Session 2024-25
Subject - Mathematics
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. $\sin \frac{\pi}{12}$ = ? **[1]**

a) $\frac{(\sqrt{3}+1)}{2\sqrt{2}}$ b) $\frac{(\sqrt{3}-1)}{2\sqrt{2}}$

c) $\frac{(2\sqrt{3}+1)}{3\sqrt{2}}$ d) $\frac{-(\sqrt{3}-1)}{2\sqrt{2}}$

2. If n(A) = 10, n(B) = 6 and n (C) = 5 for three disjoint sets A, B and C, then $n(A \cup B \cup C)$ = **[1]**

a) 11 b) 21

c) 1 d) 9

3. The mean of the series $x_1, x_2, ..., x_n$ is $\overline{X}$. If x_2 is replaced by λ, then what is the new mean? **[1]**

a) $\frac{\bar{X}-x_2-\lambda}{n}$ b) $\frac{n\bar{X}-x_2-\lambda}{n}$

c) $\frac{\bar{X}-x_2+\lambda}{n}$ d) $\overline{X}-x_2+\lambda$

4. If f(x) = x sinx, then f' $\left(\frac{\pi}{2}\right)$ is equal to **[1]**

a) 1 b) $\frac{1}{2}$

c) -1 d) 0

5. The coordinates of the foot of perpendicular from (0, 0) upon the line x + y = 2 are **[1]**

a) (1, 1) b) (1, -2)

c) (-1, 2) d) (1, 2)

6. The length of the foot of perpendicular drawn from the point P (3, 4, 5) on y-axis is **[1]**

a) $\sqrt{34}$ b) 10

c) $\sqrt{113}$ d) $5\sqrt{2}$

7. Mark the correct answer for $3i^{34} + 5i^{27} - 2i^{38} + 5i^{41}$ = ? **[1]**

a) 1 b) -1

c) -4i d) 10i

8. A fair dice is rolled n times. The number of all the possible outcomes is **[1]**

a) 6n b) n^6

c) 6^n d) 6+n

9. If y = $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$ + ..., then $\frac{dy}{dx}$ = **[1]**

a) y^2 b) y + 1

c) y d) y - 1

10. If $\frac{3\pi}{4} < \alpha < \pi$,then $\sqrt{2\cot\alpha + \frac{1}{\sin^2\alpha}}$ is equal to **[1]**

a) - 1 + cot α b) - 1 - cot α

c) 1 - cot α d) 1 + cot α

11. Each set X_r contains 5 elements and each set Y_r contains 2 elements and $\bigcup_{r=1}^{20} x_r = S = \bigcup_{r=1}^{n} Y_r$. If each element of S belong to exactly 10 of the X_r's and to exactly 4 of the Y_r's, then n is **[1]**

a) 10 b) 20

c) 50 d) 100

12. In the expansion of $(x + a)^n$, if the sum of odd terms be P and the sum of even terms be Q, then 4PQ = ? **[1]**

a) $(x + a)^n - (x - a)^n$ b) $(x + a)^{2n} - (x - a)^{2n}$

c) $(x + a)^n + (x - a)^n$ d) $(x + a)^{2n} + (x - a)^{2n}$

13. If $\left(1 - x + x^2\right)^n = a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$, then $a_0 + a_2 + a_4 + \ldots + a_{2n}$ equals. **[1]**

a) $3^n + \frac{1}{2}$ b) $\frac{3^n+1}{2}$

c) $\frac{3^n-1}{2}$ d) $\frac{1-3^n}{2}$

14. If x is a real number and $|x| < 3$, then **[1]**

a) - 3 < x < 3 b) $x \geq -3$

c) $x \geq 3$ d) $-3 \leq x \leq 3$

15. Which of the following is a set? **[1]**

A. A collection of vowels in English alphabets is a set.

B. The collection of most talented writers of India is a set.

C. The collection of most difficult topics in Mathematics is a set.

D. The collection of good cricket players of India is a set.

a) B b) D

c) A d) C

16. If 3 sin x + 4 cos x = 5, then 4 sin x - 3 cos x = **[1]**

a) 1

b) 5

c) 3

d) 0

17. $\lim_{x \to \pi} \frac{\sin x}{x-\pi}$ is equal to **[1]**

a) 1

b) -1

c) 2

d) -2

18. The value of $\left({}^7C_0 + {}^7C_1\right)$ + $\left({}^7C_1 + {}^7C_2\right)$ + ... + $\left({}^7C_6 + {}^7C_7\right)$ is **[1]**

a) $2^8 - 2$

b) $2^8 - 1$

c) $2^7 - 1$

d) 2^8

19. **Assertion (A):** The collection of all natural numbers less than 100' is a set. **[1]**

Reason (R): A set is a well-defined collection of the distinct objects.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If the sum of first two terms of an infinite GP is 5 and each term is three times the sum of the succeeding terms, then the common ratio is $\frac{1}{4}$. **[1]**

Reason (R): In an AP 3, 6, 9, 12 .... the 10th term is equal to 33.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Let f, g be two real functions defined by $f(x) = \sqrt{x+1}$ and $g(x) = \sqrt{9-x^2}$ Then describe each of the following functions: f + g. **[2]**

OR

Find the domain and the range of the real function: f(x) = $\frac{|x-4|}{x-4}$

22. Evaluate $\lim_{x \to 1} \frac{x^3-1}{x-1}$. **[2]**

23. If A and B are two events associated with a random experiment such that P(A) = 0.25, P(B) = 0.4 and P(A or B) = 0.5, find the values of **[2]**

i. P(A and B)

ii. P(A and $\bar{B}$)

OR

Check whether the probabilities P(A) and P(B) are consistently defined P(A) = 0.5, P(B) = 0.4, $P(A \cup B) = 0.8$

24. In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group? **[2]**

25. Find the length of perpendicular from the origin to the lines 7x + 24y = 50. **[2]**

Section C

26. In how many ways can six persons be seated in a row? **[3]**

27. Find the point in yz-plane which is equidistant from the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2). **[3]**

28. Find $(x + 1)^6 + (x - 1)^6$. Hence or otherwise evaluate $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$ **[3]**

OR

Using g binomial theorem, expand $\left\{(x+y)^5 + (x-y)^5\right\}$ and hence find the value of $\{(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5\}$

29. Evaluate the following limits: $\lim_{x\to 0} \frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$. **[3]**

OR

Differentiate e^{ax+b} from first principle.

30. The sum of three numbers a, b, c in A.P. is 18. If a and b are each increased by 4 and c is increased by 36, the new numbers form a G.P. Find a, b, c. **[3]**

OR

If A.M. and G.M. of roots of a quadratic equation are 8 and 5 respectively then obtain the quadratic equation.

31. Are the E = {x : x ∈ Z, $x^2 \leq 4$} and F = {x : x ∈ Z, $x^2 = 4$} pairs of equal set? **[3]**

Section D

32. The mean and standard deviation of 100 observations were calculated as 40 and 5.1, respectively by a student who took by mistake 50 instead of 40 for one observation. What are the correct mean and standard deviation? **[5]**

33. Fine the lengths major and minor axes, coordinates of the vertices, coordinates of the foci, eccentricity, and length of the latus rectum of the ellipse $25x^2 + 4y^2 = 100$. **[5]**

OR

A visitor with sign board 'DO NOT LITTER' is moving on a circular path in an exhibition. During the movement he stops at points represented by (3, - 2) and (-2, 0). Also, centre of the circular path is on the line 2x - y = 3. What is the equation of the path? What message he wants to give to the public?

34. Solve the following system of linear inequalities **[5]**

$\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4}$ and $\frac{7x-1}{3} - \frac{7x+2}{6} > x$.

35. Prove that: $\cot x + \cot\left(\frac{\pi}{3}+x\right) + \cot\left(\frac{2\pi}{3}+x\right) = 3\cot 3x$. **[5]**

OR

If A + B + C= π, prove that $\frac{\sin 2A+\sin 2B+\sin 2C}{\sin A+\sin B+\sin C} = 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Method to Find the Sets When Cartesian Product is Given

For finding these two sets, we write first element of each ordered pair in first set say A and corresponding second element in second set B (say).

Number of Elements in Cartesian Product of Two Sets

If there are p elements in set A and q elements in set B, then there will be pq elements in A × B i.e. if n(A) = p and n(B) = q, then n(A × B) = pq.

i. The Cartesian product A × A has 9 elements among which are found (-1, 0) and (0, 1). Find the set A and the remaining elements of A × A. (1)

ii. A and B are two sets given in such a way that A × B contains 6 elements. If three elements of A × B are (1, 3), (2, 5) and (3, 3), then find the remaining elements of A × B. (1)

iii. If the set A has 3 elements and set B has 4 elements, then find the number of elements in A × B. (2)

OR

If A × B = {(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)}. Find A and B. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

On her vacation, Priyanka visits four cities. Delhi, Lucknow, Agra, Meerut in a random order.


Meerut


New Delhi


Agra


Lucknow

i. What is the probability that she visits Delhi before Lucknow? (1)

ii. What is the probability she visit Delhi before Lucknow and Lucknow before Agra? (1)

iii. What is the probability she visits Delhi first and Lucknow last? (2)

OR

What is the probability she visits Delhi either first or second? (2)

38. Two complex numbers $Z_1 = a + ib$ and $Z_2 = c + id$ are said to be equal, if a = c and b = d. **[4]**

i. If (x + iy)(2 - 3i) = 4 + i then find the value of (x, y). (1)

ii. If $\frac{(1+i)^2}{2-i} = x + iy$, then find the value of x + y. (1)

iii. If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then find the values of a and b. (2)

OR

If (a - 2, 2b + 1) = (b - 1, a + 2), then find the real values of a and b. (2)

Solution

Section A

1.

(b) $\frac{(\sqrt{3}-1)}{2\sqrt{2}}$

Explanation: $\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right) = \frac{(\sqrt{3}-1)}{2\sqrt{2}}$

2.

(b) 21

Explanation: Since A, B, C are disjoint

$\therefore n(A \cup B \cup C)$ = n(a) + n(B) + n(C)

= 10 + 6 + 5 = 21

3.

(b) $\frac{n\bar{X}-x_2-\lambda}{n}$

Explanation: We know, $\bar{X} = \frac{x_1+x_2+\ldots+x_n}{n} \Rightarrow$ $x_1 + x_2 + \ldots + x_n = n\overline{X}$

$\Rightarrow x_1 + x_2 + \ldots + x_n = n\overline{X} - x_2$

$\Rightarrow x_1 + x_3 + \ldots + x_n + \lambda = n\overline{X} - x_2 + \lambda$

$\Rightarrow$ Mean = $\frac{\text{Sum of all values}}{\text{Total number of values}} = \frac{x_1+x_3+\ldots+x_n+\lambda}{n}$

$= \frac{n\bar{X}-x_2-\lambda}{n}$

4. **(a)** 1

Explanation: f′ (x) = x cosx + sinx

So, $f'\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2} = 1$

5. **(a)** (1, 1)

Explanation: The equation of the line perpendicular to the given line is x - y + k = 0

Since it passes through the origin,

0 - 0 + k = 0

Therefore, k = 0

Hence the equation of the line is x - y = 0

On solving these two equations we get x = 1 and y = 1

The point of intersection of these two lines is (1, 1)

Hence the coordinates of the foot of the perpendicular is (1, 1)

6. **(a)** $\sqrt{34}$

Explanation: Let l be the foot of the perpendicular from point P on the y-axis. Therefore, its x and z-coordinates are zero, i.e., (0, 4, 0). Therefore, the distance between the points (0, 4, 0) and (3, 4, 5) is $\sqrt{9+25}$ = $\sqrt{34}$.

7.

(b) -1

Explanation: $3i^{34} + 5i^{27} - 2i^{38} + 5i^{41} = 3\times(i^4)^8\times i^2 + 5\times(i^4)^6\times i^3 - 2\times(i^4)^9\times i^2 + 5\times(i^4)^{10}\times i$

$= 3\times1\times(-1) + 5\times1\times(-i) - 2\times1\times(-1) + 5\times1\times i$

= -3 - 5i + 2 + 5i = -1

8.

(c) 6^n

Explanation: Each time there are 6 possibilities, therefore for n times there are 6^n possibilities.

9.

(c) y

Explanation: y = 1 + $\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}$ + ...

Differentiating both sides with respect to x, we get $\frac{dy}{dx}=\frac{d}{dx}\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^2}{3!}+\ldots\right)$

= $\frac{d}{dx}(1)+\frac{d}{dw}\left(\frac{x}{11}\right)+\frac{d}{dw}\left(\frac{x^2}{2!}\right)+\frac{d}{dw}\left(\frac{x^3}{3!}\right)+\frac{d}{dx}\left(\frac{x^4}{4!}\right)$ + ...

= $\frac{d}{dx}(1)+\frac{1}{1!}\frac{d}{dx}(x)+\frac{1}{2!}\frac{d}{dw}\left(x^2\right)+\frac{1}{3!}\frac{d}{dw}\left(x^3\right)+\frac{1}{4!}\frac{d}{dw}\left(x^4\right)$ + ...

= 0 + $\frac{1}{1!}\times$ 1 + $\frac{1}{2!}\times 2\alpha+\frac{1}{3!}\times 3\alpha^2+\frac{1}{4!}\times 4\alpha^3$ + ... (y = $\alpha^2\Rightarrow\frac{dy}{\partial\alpha}$ = nα^{n-1})

= 1 + $\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}$ + ... $\left[\frac{x}{n!}=\frac{1}{(n-1)!}\right]$

= y

$\therefore\frac{dy}{dx}$ = y

10.

(b) - 1 - cot α

Explanation: We have:

$\sqrt{2\cot\alpha+\frac{1}{\sin^2\alpha}}$

$=\sqrt{\frac{2\cos\alpha}{\sin\alpha}+\frac{1}{\sin^2\alpha}}$

$=\sqrt{\frac{2\sin\alpha\cos\alpha+1}{\sin^2\alpha}}$

$=\sqrt{\frac{2\sin\alpha\cos\alpha+\sin^2\alpha+\cos^2\alpha}{\sin^2\alpha}}$

$=\sqrt{\frac{(\sin\alpha+\cos\alpha)^2}{\sin^2\alpha}}$

$=\sqrt{(1+\cot\alpha)^2}$

$=|1+\cot\alpha|$

$=-(1+\cot\alpha)$ [When $\frac{3\pi}{4}<\alpha<\pi,\cot\alpha<-1\ \Rightarrow\cot\alpha$ + 1 < 0]

= - 1 - cot α

11.

(b) 20

Explanation: The correct answer is (B)

Since, n(X_r) = 5, $\bigcup_{r=1}^{20}X_r$ = S, we obtain n(S) = 100

But each element of S belong to exactly 10 of the X 's

Thus, $\frac{100}{10}$ = 10 are the number of distinct elements in S.

Also each element of S belong to exactly 4 of the Y_r's and each $Y_{r's}$contain 2 elements. If S has n number of Y_r in it.

Then $\frac{2n}{4}$ = 10

which gives n = 20

12.

(b) $(x + a)^{2n} - (x - a)^{2n}$

Explanation: P + Q = $(x + a)^n$ and P - Q = $(x - a)^n$

$\Rightarrow$ 4PQ = $(P + Q)^2 - (P - Q)^2 = (x + a)^{2n} - (x - a)^{2n}$

13.

(b) $\frac{3^n+1}{2}$

Explanation: $\left(1-x+x^2\right)^n=a_0+a_1x+a_2x^2+\ldots+a_{2n}x^{2n}$ ..(1)

Put x=1 in (1),we get

$1=a_0+a_1+a_2+a_3+\ldots+a_{2n}$ ..(2)

Put x=-1 in(1),we get

$3^n=a_0-a_1+a_2-a_3+\ldots+a_{2n}$ ..(3)

Adding(1) and(2),we get

$3^n+1=2\left(a_0+a_2+a_4+\ldots+a_{2n}\right)$

Thus, $a_0+a_2+a_4+\ldots+a_{2n}=\frac{3^n+1}{2}$

14. **(a)** - 3 < x < 3

Explanation: We have $|x|<a\Leftrightarrow -a<x<a$

15.

(c) A

Explanation: The set is {a, e, i, o, u}

16.

(d) 0

Explanation: 3 sinx + 4 cosx = 5

$\frac{3}{5}\sin x + \frac{4}{5}\cos x = 1$

Let $\cos\alpha = \frac{3}{5}$ and $\sin\alpha = \frac{4}{5}$

$\therefore \cos\alpha\sin x + \sin\alpha\cos x = 1$

$\Rightarrow \sin(\alpha + x) = \sin\frac{\pi}{2}$

$\Rightarrow \alpha + x = \frac{\pi}{2}$

$\Rightarrow x = \frac{\pi}{2} - \alpha$ .... (i)

We have to find the value of 4 sin x - 3 cos x

$4sin(\frac{\pi}{2} - \alpha) - 3cos(\frac{\pi}{2} - \alpha)$ .... From eq. (i)

$= 4\cos\alpha - 3\sin\alpha$

$= 4 \times \frac{3}{5} - 3 \times \frac{4}{5}\left(\because \cos\alpha = \frac{3}{5} \text{ and } \sin\alpha = \frac{4}{5}\right)$

0

17.

(b) -1

Explanation: Given, $\lim_{x\to\pi}\frac{\sin x}{x-\pi} = \lim_{x\to\pi}\frac{\sin(\pi-x)}{-(\pi-x)}$

$= -1\left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1 \text{ and } \pi - x \to 0 \Rightarrow x \to \pi\right]$

18. **(a)** 2^8 - 2

Explanation: $\left({}^7C_0 + {}^7C_1\right) + \left({}^7C_1 + {}^7C_2\right) + \left({}^7C_2 + {}^7C_3\right) + \left({}^7C_4 + {}^7C_5\right) + \left({}^7C_5 + {}^7C_6\right) + \left({}^7C_6 + {}^7C_7\right)$

= $1 + 2 \times {}^7C_1 + 2 \times {}^7C_2 + 2 \times {}^7C_3 + 2 \times {}^7C_5 + 2 \times {}^7C_6 + 1$

= $1 + 2 \times {}^7C_1 + 2 \times {}^7C_2 + 2 \times {}^7C_3 + 2 \times {}^7C_2 + 2 \times {}^7C_6 + 1$

= $2 + 2^2\left({}^7C_1 + {}^7C_2 + {}^7C_3\right)$

= $2 + 2^2\left(7 + \frac{7}{2} \times 6 + \frac{7}{3} \times \frac{6}{2} \times 5\right)$

= 2 + 252

= 254

= 2^8 - 2

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Assertion The collection of all natural numbers less than 100', is a well-defined collection. So, it is a set.

20.

(c) A is true but R is false.

Explanation: Assertion Let a be the first term and r(|r| < 1) be the common ratio of the GP.

$\therefore$ The GP is a, ar, ar^2,...

According to the question,

$T_1 + T_2 = 5 \Rightarrow a + ar = 5 \Rightarrow a(1 + r) = 5$

and $T_n = 3(T_{n+1} + T_{n+2} + T_{n+3} + ...)$

$\Rightarrow ar^{n-1} = 3(ar^n + ar^{n+1} + ar^{n+2} + ...)$

$\Rightarrow ar^{n-1} = 3ar^n(1 + r + r^2 + ...)$

$\Rightarrow 1 = 3r(\frac{1}{1-r})$

$\Rightarrow 1 - r = 3r$

$\Rightarrow r = \frac{1}{4}$

Reason: Given, 3, 6, 9, 12 ...

Here, a = 3, d = 6 - 3 = 3

$\therefore T_{10} = a + (10 - 1)d$

= 3 + 9 × 3

= 3 + 27 = 30

Section B

21. According to the question , we can state,

We know the square of a real number is never negative.

Clearly, f(x) takes real values only when x + 1 $\geq$ 0

= x > -1

$\therefore x \in [-1, \infty)$

Thus, domain of f = $(-1, \infty)$

Similarly, g(x) takes real values only when 9 - $x^2 \geq 0$

= 9 > x^2

= x^2 < 9

= x^2 - 9 < 0

= x^2 - 32 < 0

= (x + 3) (x - 3) < 0

= x $\geq$ -3 and x < 3

$x \in [-3, 3]$

Thus, domain of g = $= [-3, 3]$

i.f + g

We know (f + g)(x) = f(x) + g(x)

$\therefore (f+g)(x) = \sqrt{x+1} + \sqrt{9-x^2}$

Domain of f + g = Domain of f ∩ Domain of g

= Domain of f + g = $[-1, \infty) \cap [-3, 3]$

Domain of f + g = [-1, 3]

Thus, f + g : [-1, 3] R is given by $(f+g)(x) = \sqrt{x+1} + \sqrt{9-x^2}$

OR

Here we have, f(x) = $\frac{|x-4|}{x-4}$

We need to find where the function is defined.

To find the domain of the function f(x) we need to equate the denominator of the function to 0

Therefore,

x - 4 = 0 or x = 4

It means that the denominator is zero when x = 4

So, the domain of the function is the set of all the real numbers except 4

The domain of the function, $D_{\{f(x)\}}$ = (-∞, 4) ∪ (4, ∞)

The numerator is an absolute function of the denominator.

So, for any value of x from the domain set, we always get either +1 or -1 as the output.

So, the range of the function is a set containing -1 and +1

Therefore, the range of the function, $R_{f(x)}$ = {-1, 1}

22. Given, $\lim_{x \to 1} \frac{x^3-1}{x-1}$

If we put x = 1, then expression $\frac{x^3-1}{x-1}$ becomes the indeterminate form $\frac{0}{0}$. Therefore, (x - 1) is a common factor of (x^3 - 1) and (x - 1).

Factorising the numerator and denominator, we have

$\lim_{x \to 1} \frac{x^3-1}{x-1}$ [$\frac{0}{0}$ form]

$= \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x-1)}$

= $\lim_{x \to 1}$ (x^2 + x + 1) = 1^2 + 1 + 1 = 3

23. i. It is given that

: P(A) = 0.25, P(A or B) = 0.5 and P(B) = 0.4

To find : P(A and B)

Formula used : P(A or B) = P(A) + P(B) - P(A and B)

Substituting the value in the above formula we get,
0.5 = 0.25 + 0.4 - P(A and B)
0.5 = 0.65 - P(A and B)
P(A and B) = 0.65 - 0.5
P(A and B) = 0.15

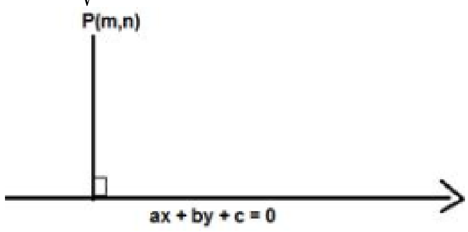
ii. Given : P(A) = 0.25, P(A and B) = 0.15 (from part (i))
To find : P(A and $\bar{B}$)
Formula used : P(A and $\bar{B}$) = P(A) - P(A and B)
Substituting the value in the above formula we get,
P(A and $\bar{B}$) = 0.25 - 0.15
P(A and $\bar{B}$) = 0.10
P(A and $\bar{B}$) = 0.10

OR

Given that P(A) = 0.5, P(B) = 0.4 and $P(A \cup B) = 0.8$
Applying the general addition rule,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\therefore\ 0.8 = 0.5 + 0.4 - P(A \cap B)$
$\Rightarrow\ P(A \cap B) = 0.9 - 0.8 = 0.1$
$\therefore\ P(A \cap B) < P(A)$ and $P(A \cap B) < P(B)$
Thus the given probabilities are consistently defined.

24. Let H be the set of students who know Hindi and E be the set of students who know English.
Here n(H) = 100, n(E) = 50 and $n(H \cap E) = 25$
We know that $n(H \cup E) = n(H) + n(E) - n(H \cap E)$
= 100 + 50 - 25 = 125.

25. Here, it is given: Point (0,0) and line 7x + 24y = 50
We have to find: The length of the perpendicular from the origin to the line 7x + 24y = 50
We know that the length of the perpendicular from P (m,n) to the line ax + by + c = 0 is given by,
$D = \frac{|am+bn+c|}{\sqrt{a^2+b^2}}$



The given equation of the line is 7x + 24y - 50=0
Here m= 0 and n = 0, a = 7, b = 24, c = -50
$D = \frac{|7(0)+24(0)-50|}{\sqrt{7^2+24^2}}$
$D = \frac{|0+0-50|}{\sqrt{49+576}} = \frac{|-50|}{\sqrt{625}} = \frac{|-50|}{25} = \frac{50}{25} = 2$
D = 2
Therefore, the length of perpendicular from the origin to the line 7x + 24y = 50 is 2 units.

Section C

26. Given: Six persons are to be arranged in a row.
Assume six seats, now in the first seat, any one of six members can be seated, so the total number of possibilities is $^6C_1$
Similarly, in the second seat, any one of five members can be seated, so the total number of possibilities is $^5C_1$
In the third seat, any one of four members can be seated, so the total number of possibilities is $^4C_1$
In the fourth seat, any one of three members can be seated, so the total number of possibilities is $^3C_1$
In the fifth seat, any one of two members can be seated, so the total number of possibilities is $^2C_1$

In the sixth seat, only one remaining person can be seated, so the total number of possibilities is ${}^1C_1$

Hence the total number of possible outcomes = ${}^6C_1 \times {}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1 = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

27. The general point on yz plane is D(0, y, z).

Consider this point is equidistant to the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2).

∴ AD = BD

$\sqrt{(0-3)^2+(y-2)^2+(z+1)^2} = \sqrt{(0-1)^2+(y+1)^2+(z-0)^2}$

Squaring both sides,

$(0 - 3)^2 + (y - 2)^2 + (z + 1)^2 = (0 - 1)^2 + (y + 1)^2 + (z - 0)^2$

$9 + y^2 - 4y + 4 + z^2 + 2z + 1 = 1 + y^2 + 2y + 1 + z^2$

$-6y + 2z + 12 = 0$ ….(1)

Also, AD = CD

$\sqrt{(0-3)^2+(y-2)^2+(z+1)^2} = \sqrt{(0-2)^2+(y-1)^2+(z-2)^2}$

Squaring both sides,

$(0 - 3)^2 + (y - 2)^2 + (z + 1)^2 = (0 - 2)^2 + (y - 1)^2 + (z - 2)^2$

$9 + y^2 - 4y + 4 + z^2 + 2z + 1 = 4 + y^2 - 2y + 1 + z^2 - 4z + 4$

$-2y + 6z + 5 = 0$ ….(2)

By solving equation (1) and (2) we get

$y = \frac{31}{16}$ $z = \frac{-3}{16}$

The point which is equidistant to the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2) is $(\frac{31}{16}, \frac{-3}{16})$.

28. $(x + 1)^6 + (x - 1)^6 == [{}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6]$

$+[{}^6C_0x^6 + {}^6C_1x^5(-1) + {}^6C_2x^4(-1)^2 + {}^6C_3x^3(-1)^3 + {}^6C_4x^2(-1)^4 + {}^6C_5x(-1)^5 + {}^6C_6(-1)^6]$

$= [x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1] + [x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1]$

$= 2x^6 + 30x^4 + 30x^2 + 2$

$= 2(x^6 + 15x^4 + 15x^2 + 1)$

Putting $x = \sqrt{2}$

$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 = 2[(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1]$

$= 2\,[8 + 15 \times 4 + 15 \times 2 + 1]$

= 2 [8 + 60 + 30 + 1]

$= 2 \times 99 = 198$

OR

$We\ have$

$(x+y)^5 + (x-y)^5 = 2\,[{}^5C_0\,x^5 + {}^5C_2\,x^3y^2 + {}^5C_4\,x^1y^4]$

$= 2\,(x^5 + 10x^3y^2 + 5xy^4$

$Putting\ x = \sqrt{2}\ and\ y = 1,\ we\ get$

$(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5 = 2\left[(\sqrt{2})^5 + 10(\sqrt{2})^3 + 5\sqrt{2}\right]$

$= 2\,[4\sqrt{2} + 20\sqrt{2} + 5\sqrt{2}]$

$= 58\sqrt{2}$

29. Given: $\lim_{x\to 0} \frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$

Rationalizing the given equation,

$= \lim_{x\to 0} \frac{2x}{(\sqrt{a+x}-\sqrt{a-x})} \frac{(\sqrt{a+x}+\sqrt{a-x})}{(\sqrt{a+x}+\sqrt{a-x})}$

Formula: $(a + b)(a - b) = a^2 - b^2$

$= \lim_{x\to 0} \frac{2x(\sqrt{a+x}+\sqrt{a-x})}{a+x-a+x}$

$= \lim_{x\to 0} \frac{2x(\sqrt{a+x}+\sqrt{a-x})}{2x}$

$= \lim_{x\to 0} \frac{(\sqrt{a+x}+\sqrt{a-x})}{1}$

Now we can see that the indeterminant form is removed, so substituting x as 0

Therefore, $\lim\limits_{x \to 0} \frac{2x}{\sqrt{a+x}-\sqrt{a-x}} = \sqrt{a}+\sqrt{a}=2\sqrt{a}$

OR

We need to find derivative of $f(x) = e^{ax + b}$

Derivative of a function f(x) is given by

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

$\therefore$ derivative of $f(x) = e^{ax + b}$ is given as

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{e^{a(x+h)+b}-e^{ax+b}}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{e^{ax+b}e^{ah}-e^{ax+b}}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{e^{ax+b}\left(e^{ah}-1\right)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} e^{ax+b} \times \lim\limits_{h \to 0} \frac{e^{ah}-1}{h}$

As one of the limits $\times \lim\limits_{h \to 0} \frac{e^{ah}-1}{h}$ can't be evaluated by directly putting the value of h as it will take $\frac{0}{0}$ form.

So we need to take steps to find its value.

$\Rightarrow f'(x) = \lim\limits_{h \to 0} e^{ax+b} \times \lim\limits_{h \to 0} \frac{e^{ah}-1}{ah} \times a$

Use the formula: $\lim\limits_{x \to 0} \frac{e^{x}-1}{x} = \log_e e = 1$

$\Rightarrow f'(x) = e^{ax + b} \times (a)$

$\Rightarrow f'(x) = ae^{ax + b}$

Hence,

Derivative of $f(x) = e^{ax + b} = ae^{ax + b}$

30. Let the first term of the A.P. be a and the common difference be d.

$\therefore$ a = a, b = a + d and c = a + 2d

a + b + c = 18

$\Rightarrow$ 3a + 3d = 18

$\Rightarrow$ a + d = 6 ...(i)

Now, according to the question, a + 4, a + d + 4 and a + 2d + 36 are in G.P.

$\therefore (a + d + 4)^2 = (a + 4)(a + 2d + 36)$

$\Rightarrow (6 - d + d + 4)^2 = (6 - d + 4)(6 - d + 2d + 36)$ [using(i)]

$\Rightarrow (10)^2 = (10 - d)(42 + d)$

$\Rightarrow 100 = 420 + 10d - 42d - d^2$

$\Rightarrow d^2 + 32d - 320 = 0$

$\Rightarrow$ (d + 40)(d - 8) = 0

$\Rightarrow$ d = 8, -40

Now, substituting d = 8, -40 in equation (i), we obtain, a = -2, 46, respectively.

For a = -2 and d = 8, we obtain

a = -2, b = 6, c = 14

And for a = 46 and d = -40, we obtain

a = 46, b = 6, c = -34

OR

Let a and b be the roots of required quadratic equation.

Then A.M. = $\frac{a+b}{2}$ = 8

$\Rightarrow$

a + b = 16

And G.M. = $\sqrt{ab}$ = 5

$\Rightarrow$ ab = 25

Now, Quadratic equation x^2 - (Sum of roots) x + (Product of roots) = 0

$\Rightarrow x^2 - (a + b)x + ab = 0$

$\Rightarrow x^2 - 16x + 25 = 0$

Therefore, required equation is $x^2 - 16x + 25 = 0$

31. We know two sets A and B are said to be equal if they have exactly the same elements & we write A = B

We have, $E = \{x : x \in Z, x^2 \leq 4\}$

Here, $x \in Z$ and $x^2 \leq 4$

If x = -2, then $x^2 = (-2)^2 = 4 = 4$

If x = -1, then $x^2 = (-1)^2 = 1 < 4$

If x = 0, then $x^2 = (0)^2 = 0 < 4$

If x = 1, then $x^2 = (1)^2 = 1 < 4$

If x = 2, then $x^2 = (2)^2 = 4 = 4$

Therefore, E = {-2, -1, 0, 1, 2}

and $F = \{x : x \in Z, x^2 = 4\}$

Here, $x \in Z$ and $x^2 = 4$

If x = -2, then $x^2 = (-2)^2 = 4 = 4$

If x = 2, then $x^2 = (2)^2 = 4 = 4$

Therefore, F = {-2, 2}

$\therefore E \neq F$ because the elements in the both the sets are not equal.

Section D

32. We have, n = 100, $\overline{x}$ = 40 and σ = 5.1

$\therefore \overline{x} = \frac{1}{n}\Sigma x_i$

$\Rightarrow \Sigma x_i = n\overline{x} = 100 \times 40 = 4000$

$\therefore$ Incorrect $\Sigma x_i = 4000$

and,

$\sigma = 5.1$

$\Rightarrow \sigma^2 = 26.01$

$\Rightarrow \frac{1}{n}\Sigma x_i^2 - (\text{mean})^2 = 26.01$

$\Rightarrow \frac{1}{100}\Sigma x_i^2 - 1600 = 26.01$

$\Rightarrow \Sigma x_i^2 = 1626.01 \times 100$

$\therefore$ Incorrect $\Sigma x_i^2 = 162601$

To correct the $\sum x_i$, we need to subtract the incorrect observation 50 and add correct observation is 40.

We have, incorrect $\Sigma x_i = 4000$

$\therefore$ Correct $\Sigma x_i = 4000 - 50 + 40 = 3990$

and,

Similarly, to obtain correct $\sum x_i^2$ we need to subtract 50^2 and add 40^2 to incorrect one.

Incorrect $\Sigma x_i^2 = 162601$

$\therefore$ Correct $\Sigma x_i^2 = 162601 - 50^2 + 40^2 = 161701$

Now, Correct mean = $\frac{3990}{100}$ = 39.90

Correct variance = $\frac{1}{100}$ (Correct Σx_i^2) - (Correct mean)^2

$\Rightarrow$ Correct variance = $\frac{161701}{100} - \left(\frac{3990}{100}\right)^2$

$\Rightarrow$ Correct variance = $\frac{161701 \times 100 - (3990)^2}{(100)^2}$

$\Rightarrow$ Correct variance = $\frac{16170100 - 15920100}{10000}$ = 25

$\therefore$ Correct standard deviation = $\sqrt{25}$ = 5

33. Given that:

$25x^2 + 4y^2 = 100$

after divide by 100 to both the sides, we get

$\frac{25}{100}x^2 + \frac{4}{100}y^2 = 1 \Rightarrow \frac{x^2}{4} + \frac{y^2}{25} = 1$ ... (i)

Now, above equation is of the form,

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ ... (ii)

Comparing eq. (i) and (ii), we get

$a^2 = 25$ and $b^2 = 4 \Rightarrow a = \sqrt{25}$ and $b = \sqrt{4} \Rightarrow a = 5$ and $b = 2$

i. Length of major axes

$\therefore$ Length of major axes = 2a = 2 $\times$ 5 = 10 units

ii. Length of minor axes

Length of minor axes = 2b = 2 $\times$ 2 = 4 units

iii. Coordinates of the vertices

$\therefore$ Coordinates of the vertices = (0, a) and (0, -a) = (0, 5) and (0, -5)

iv. Coordinates of the foci

As we know that,

Coordinates of foci $= (0, \pm c)$ where $c^2 = a^2 - b^2$

Now

$c^2 = 25 - 4 \Rightarrow c^2 = 21 \Rightarrow c = \sqrt{21}$ ... (iii)

$\therefore$ Coordinates of foci $= (0, \pm\sqrt{21})$

v. Eccentricity

As we know that, Eccentricity $= \frac{c}{a} \Rightarrow e = \frac{\sqrt{21}}{5}$

vi. Length of the Latus Rectum

As we know that Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2\times(2)^2}{5} = \frac{8}{5}$

OR

Let the equation of circle whose centre (- g, - f) be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Since, is passes through points (3, - 2) and (- 2, 0)

$\therefore (3)^2 + (-2)^2 + 2g(3) + 2f(-2) + c = 0$

and $(-2)^2 + (0)^2 + 2g(-2) + 2f(0) + c = 0$

$\Rightarrow 9 + 4 + 6g - 4f + c = 0$

and $4 + 0 - 4g + 0 + c = 0$

$\Rightarrow 6g - 4f + c = -13$

and $c = 4g - 4$ ...(ii)

$\therefore 6g - 4f + (4g - 4) = -13$

$\Rightarrow 10g - 4f = -9$ ...(iii)

Also, centre (- g, - f) lies on the line 2x - y = 3

$\therefore -2g + f = 3$ ...(iv)

On solving Eqs. (iii) and (iv), we get

$g = \frac{3}{2}$ and $f = 6$

On putting the values of g and f in Eq. (ii), we get

$c = 4\left(\frac{3}{2}\right) - 4 = 6 - 4 = 2$

On putting the values of g, f and c in Eq. (i), we get

$x^2 + y^2 + 2\left(\frac{3}{2}\right)x + 2(6)x + 2 = 0$

$\Rightarrow x^2 + y^2 + 3x + 12x + 2 = 0$

which is the required equation of the path

The message which he wants to give to the public is 'Keep your place clean'.

34. We have, $\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4}$ ... (i)

and $\frac{7x-1}{3} - \frac{7x+2}{6} > x$ ... (ii)

From inequality (i), we get

$\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4} \Rightarrow \frac{16x-27}{12} < \frac{4x+3}{4}$

⇒ 16x - 27 < 12x + 9 [multiplying both sides by 12]

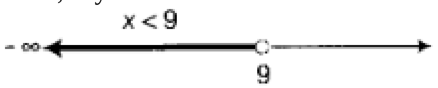
⇒ 16x - 27 + 27 < 12x + 9 + 27 [adding 27 on both sides]

⇒ 16x < 12x +36

⇒ 16x - 12x < 12x + 36 - 12x [subtracting 12x from bot sides]

⇒ 4x < 36 ⇒ x < 9 [dividing both sides by 4]

Thus, any value of x less than 9 satisfies the inequality. So, the solution of inequality (i) is given by $x \in (-\infty, 9)$



From inequality (ii) we get,

$\frac{7x-1}{3} - \frac{7x+2}{6} > x \Rightarrow \frac{14x-2-7x-2}{6} > x$

⇒ 7x - 4 > 6x [multiplying by 6 on both sides]

⇒ 7x - 4 + 4 > 6x + 4 [adding 4 on both sides]

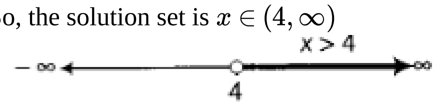
⇒ 7x > 6x + 4

⇒ 7x - 6x > 6x + 4 - 6x [subtracting 6x from both sides]

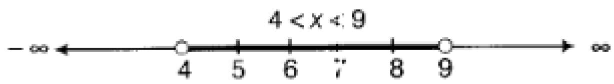
∴ x > 4

Thus, any value of x greater than 4 satisfies the inequality.

So, the solution set is $x \in (4, \infty)$



The solution set of inequalities (i) and (ii) are represented graphically on number line as given below:



Clearly, the common value of x lie between 4 and 9.

Hence, the solution of the given system is, 4 < x < 9 i.e., $x \in (4, 9)$

35. We have to prove cot x + $\cot\left(\frac{\pi}{3} + x\right)$ + $\cot\left(\frac{2\pi}{3} + x\right)$ = 3 cot 3x.

LHS = cot x + $\cot\left(\frac{\pi}{3} + x\right)$ + $\cot\left(\frac{2\pi}{3} + x\right)$

We know,

$\cot\left(\frac{2\pi}{3} + x\right) = \cot\left(\pi - \left(\frac{\pi}{3} - x\right)\right) = -\cot\left(\frac{\pi}{3} - x\right)$ ... (as - cotθ = cot (180° - θ)

Hence the above LHS becomes

$= \cot x + \cot\left(\frac{\pi}{3} + x\right) - \cot\left(\frac{\pi}{3} - x\right)$

$= \frac{1}{\tan x} + \frac{1}{\tan\left(\frac{\pi}{3}+x\right)} - \frac{1}{\tan\left(\frac{\pi}{3}-x\right)}$

$= \frac{1}{\tan x} + \left(\frac{1-\tan x \tan\frac{\pi}{3}}{\tan\frac{\pi}{3}+\tan x}\right) - \left(\frac{1+\tan x \tan\frac{\pi}{3}}{\tan\frac{\pi}{3}-\tan x}\right)$ ... [∵ $\tan(A + B) = \left(\frac{\tan A+\tan B}{1-\tan A \tan B}\right)$ and $\tan(A - B) = \left(\frac{\tan A-\tan B}{1+\tan A \tan B}\right)$]

$= \frac{1}{\tan x} + \left(\frac{1-\sqrt{3}\tan x}{\sqrt{3}+\tan x}\right) - \left(\frac{1+\sqrt{3}\tan x}{\sqrt{3}-\tan x}\right)$

$= \frac{1}{\tan x} + \left(\frac{(1-\sqrt{3}\tan x)(\sqrt{3}-\tan x)-(1+\sqrt{3}\tan x)(\sqrt{3}+\tan x)}{(\sqrt{3}+\tan x)(\sqrt{3}-\tan x)}\right)$

$= \frac{1}{\tan x} + \left(\frac{(\sqrt{3}-\tan x-3\tan x+\sqrt{3}\tan^2 x)-(\sqrt{3}+3\tan x+\tan x+\sqrt{3}\tan^2 x)}{(3-\tan^2 x)}\right)$

$= \frac{1}{\tan x} + \left(\frac{(0-4\tan x-4\tan x+0)}{(3-\tan^2 x)}\right)$

$= \frac{1}{\tan x} - \left(\frac{8\tan x}{((3-\tan^2 x))}\right)$

$= \left(\frac{(3-\tan^2 x)-8\tan^2 x}{\tan x(3-\tan^2 x)}\right) = \left(\frac{(3-\tan^2 x)-8\tan^2 x}{\tan x(3-\tan^2 x)}\right)$

$= 3\left(\frac{1-3\tan^2 x}{(3\tan x-\tan^3 x)}\right)$

$= 3 \times \frac{1}{\tan 3x}$ ... (as $\tan 3x = \frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$)

= cot 3x

LHS = RHS

Hence proved.

OR

Here it is given that, A + B + C = π

We need to prove that, $\frac{\sin 2A+\sin 2B+\sin 2C}{\sin A+\sin B+\sin C}$ = $8 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

Proof: Taking LHS, we have,

$L.H.S = \frac{\sin 2A+\sin 2B+\sin 2C}{\sin A+\sin B+\sin C}$

Where, sin 2A + sin 2B + sin 2C = 2sin A cos A + 2sin(B + C)cos(B - C)

[By using, sin A + sin B = $2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$

and sin 2A = 2sin A cos A]

Since A + B + C = π

$\Rightarrow$ B + C = 180 - A

$\therefore$ sin 2A + sin 2B + sin 2C = 2sin A cos A + 2sin(π - A)cos(B - C)

= 2sin Acos A + 2sin A cos(B - C)

= 2sin A{cos A + cos(B - C)}

(but cos A = cos { 180 - (B + C) } = - cos (B + C)

And now using

cos A - cos B = $2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{-A+B}{2}\right)$

So, sin 2A + sin 2B + sin 2C = 2sin A{2sin B sin C}

= 4sin A sin B sin C

= $32 \sin \frac{A}{2} \cos \frac{A}{2} \sin \frac{B}{2} \cos \frac{B}{2} \sin \frac{c}{2} \cos \frac{c}{2}$

Now, take denominator we have

sin A + sin B + sin C = sin A + $\left\{2 \sin\left(\frac{B+C}{2}\right) \cos\left(\frac{B-C}{2}\right)\right\}$

= sin A + $\left\{2 \sin\left(\frac{\pi-A}{2}\right) \cos\left(\frac{B-C}{2}\right)\right\}$

= sin A + $\left\{2 \cos\left(\frac{A}{2}\right) \cos\left(\frac{B-C}{2}\right)\right\}$

= $2 \sin \frac{A}{2} \cos \frac{A}{2}$ + $\left\{2 \cos\left(\frac{A}{2}\right) \cos\left(\frac{B-C}{2}\right)\right\}$

= $2 \cos \frac{A}{2} \left\{\sin \frac{A}{2} + \cos\left(\frac{B-C}{2}\right)\right\}$

= $2 \cos \frac{A}{2} \left\{\cos\left(\frac{B+C}{2}\right) + \cos\left(\frac{B-C}{2}\right)\right\}$

= $2 \cos \frac{A}{2} \left\{2 \cos\left(\frac{B}{2}\right) \cos\left(\frac{c}{2}\right)\right\}$

= $4 \cos \frac{A}{2} \cos\left(\frac{B}{2}\right) \cos\left(\frac{c}{2}\right)$

Therefore,

$L.H.S = \frac{\sin 2A+\sin 2B+\sin 2C}{\sin A+\sin B+\sin C} = \frac{32 \sin \frac{A}{2} \cos \frac{A}{2} \sin \frac{B}{2} \cos \frac{B}{2} \sin \frac{C}{2} \cos \frac{C}{2}}{4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}}$

= $8 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{c}{2}$

= R.H.S

Section E

36. i. n(A $\times$ A) = 9

$\Rightarrow$ n(A) $\subset$ n(A) = 9 $\Rightarrow$ n(A) = 3

(-1,0) $\in$ A $\times$ A $\Rightarrow$ -1 $\in$ A, 0 $\in$ A

(0,1) $\in$ A $\times$ A $\Rightarrow$ 0 $\in$ A, 1 $\in$ A

$\Rightarrow$ -1, 0, 1 $\in$ A

Also, n(A) = 3 $\Rightarrow$ A = (-1, 0, 1)

Hence, A = {-1, 0, 1}

Also, A $\times$ A = {-1, 0, 1} $\times$ {-1, 0, 1}

= {(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)}

Hence, the remaining elements of A $\times$ A are

(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0) and (1, 1).

ii. Given, (A $\times$ B) = 6 and (A $\times$ B) = {(1, 3), (2, 5), (3, 3)}

We know that Cartesian product of set A = {a, b} & B = {c, d} is A $\times$ B = {(a, c), (a, d), (b, c), (b, d)}

Therefore, A = {1, 2, 3} & B = {3, 5}

$\Rightarrow$ A $\times$ B = {(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)}

Thus, remaining elements are A $\times$ B = {(1, 5), (2, 3), (3, 5)}

iii. If the set A has 3 elements and set B has 4 elements, then the number of elements in A $\times$ B = 12

OR

Clearly, A is the set of all first entries in ordered pairs in A $\times$ B and B is the set of all second entries in ordered pairs in A $\times$ B

$\therefore$A = {a, b} and B = {1, 2, 3}

37. i. Let the Priyanka visits four cities Delhi, Lucknow, Agra, Meerut are respectively A, B, C and D. Number of way's in which Priyanka can visit four cities A, B, C and D is 4! i.e. 24

$\therefore$ n(S) = 24

Clearly, sample space for this experiment is

$$S = \left\{\begin{array}{l} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB \\ BACD, BADC, BCAD, BCDA, BDAC, BDCA \\ CABD, CADB, CBAD, CBDA, CDAB, CDBA, \\ DABC, DACB, DCAB, DCBA, DBAC, DBCA \end{array}\right\}$$

Let E_1 be the event that Priyanka visits A before B.

Then,

E_1 = {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, CABD, CADB, CDAB, DABC, DACB, DCAB}

$\Rightarrow$ n(E_1) = 12

$\therefore$ P(she visits A before B) = P(E_1) = $\frac{n(E_1)}{n(S)} = \frac{12}{24} = \frac{1}{2}$

ii. Let the Priyanka visits four cities Delhi, Lucknow, Agra, Meerut are respectively A, B, C and D. Number of way's in which Priyanka can visit four cities A, B, C and D is 4! i.e. 24

$\therefore$ n(S) = 24

Clearly, sample space for this experiment is

$$S = \left\{\begin{array}{l} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB \\ BACD, BADC, BCAD, BCDA, BDAC, BDCA \\ CABD, CADB, CBAD, CBDA, CDAB, CDBA, \\ DABC, DACB, DCAB, DCBA, DBAC, DBCA \end{array}\right\}$$

E_1 = {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, CABD, CADB, CDAB, DABC, DACB, DCAB}

$\Rightarrow$ n(E_1) = 12

$\therefore$ P(she visits A before B) $= P\left(E_1\right) = \frac{n(E_1)}{n(S)} = \frac{12}{24} = \frac{1}{2}$

iii. Let the Priyanka visits four cities Delhi, Lucknow, Agra, Meerut are respectively A, B, C and D. Number of way's in which Priyanka can visit four cities A, B, C and D is 4! i.e. 24

$\therefore$ n(S) = 24

Clearly, sample space for this experiment is

$$S = \left\{\begin{array}{l} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB \\ BACD, BADC, BCAD, BCDA, BDAC, BDCA \\ CABD, CADB, CBAD, CBDA, CDAB, CDBA, \\ DABC, DACB, DCAB, DCBA, DBAC, DBCA \end{array}\right\}$$

Let E_3 be the event that she visits A first and B last.

Then,

E_3 = {ACDB, ADCB}

n(E_3) = 2

$\because$ P(she visits A first and B last) = P(E_3)

$= \frac{n(E_3)}{n(S)} = \frac{2}{24} = \frac{1}{12}$

OR

Let the Priyanka visits four cities Delhi, Lucknow, Agra, Meerut are respectively A, B, C and D. Number of way's in which Priyanka can visit four cities A, B, C and D is 4! i.e. 24

$\therefore$ n(S) = 24

Clearly, sample space for this experiment is

$$S = \left\{\begin{array}{l} ABCD, ABDC, ACBD, ACDB, ADBC, ADCB \\ BACD, BADC, BCAD, BCDA, BDAC, BDCA \\ CABD, CADB, CBAD, CBDA, CDAB, CDBA, \\ DABC, DACB, DCAB, DCBA, DBAC, DBCA \end{array}\right\}$$

Let E_4 be the event that she visits A either first or second. Then,

E_4 = {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, CABD, CADB, DABC, DACB}

$\Rightarrow n(E_4) = 12$

Hence, P(she visits A either first or second)

$= P(E_4) = \frac{n(E_4)}{n(S)} = \frac{12}{24} = \frac{1}{2}$

38. i. $(x + iy)(2 - 3i) = 4 + i$

$2x - (3x)i + (2y)i - 3yi^2 = 4 + i$

$2x + 3y + (2y - 3x)i = 4 + i$

Comparing the real & imaginary parts,

$2x + 3y = 4$ ...(i)

$2y - 3x = 1$ ...(ii)

Solving eq (i) & eq (ii), $4x + 6y = 8$

$-9x + 6y = 3$

$13x = 5 \Rightarrow x = \frac{5}{13}$

$y = \frac{14}{13}$

$\therefore (x, y) = (\frac{5}{13}, \frac{14}{13})$

ii. $x + iy = \frac{(1+i)^2}{2-i}$

$x + iy = \frac{(1+i)^2}{2-i} = \frac{1+2i+i^2}{2-i} = \frac{2i}{2-i} = \frac{2i(2+i)}{(2-i)(2+i)} = \frac{4i+2i^2}{4-i^2}$

$= \frac{4i-2}{4+1} = \frac{-2}{5} + \frac{4i}{5}$

$\Rightarrow x = \frac{-2}{5}, y = \frac{4}{5} \Rightarrow x + y = \frac{-2}{5} + \frac{4}{5} = \frac{2}{5}$

iii. We have $\left(\frac{1-i}{1+i}\right)^{100} = a + bi$

$\Rightarrow \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^{100} = a + bi$

$\Rightarrow \left(\frac{1+i^2-2i}{1-i^2}\right)^{100} = a + bi$

$\Rightarrow \left(\frac{1-1-2i}{1+1}\right)^{100} = a + bi$

$\Rightarrow \left(\frac{-2i}{2}\right)^{100} = a + bi$

$\Rightarrow (-i)^{100} = a + bi$

$\Rightarrow i^{100} = a + bi$

$\Rightarrow (i^4)^{25} = a + bi$

$\Rightarrow (1)^{25} = a + bi$

$\Rightarrow 1 = a + bi$

$\Rightarrow 1 + 0i = a + bi$

Comparing the real and imaginary parts,

We have a = 1, b = 0

Hence (a, b) = (1, 0)

OR

Given, (a - 2, 2b + 1) = (b - 1, a + 2)

Comparing x coordinates of both the sides, we get,

a - 2 = b - 1

$\therefore$ a - b = 1 ...(1)

Comparing y coordinates of both the sides, we get,

2b + 1 = a + 2

$\therefore$ a - 2b = -1 ...(2)

Subtracting equation (2) from (1), we get,

(a - a) + (-b - (-2b)) = 1 - (-1)

$\therefore$ (-b + 2b) = 1 + 1

$\therefore$ b = 2

Put this value in equation (1), we get,

$a - 2 = 1$

$\therefore a = 3$